Technical Note
SUPERPOSING ROLLING FIELDS

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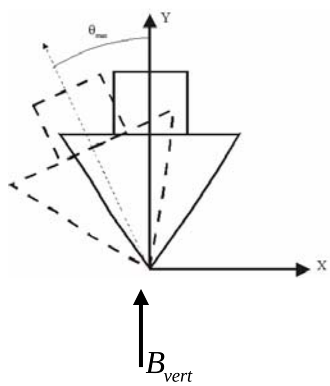
The Tech Note - Calculating Rolling Fields shows how to treat the rolling ship problem as a sequence of one DC problem and two AC steady state problems. This tech note shows how the solutions are superposed to yield the time dependent fields such as signatures, etc.

A ship rolling in the static magnetic field produced by the Earth can be modeled as a stationary ship with a rolling field. For linear materials and small maximum roll angles, the rolling field can be modeled with two, very large, orthogonal solenoids (to produce a homogenous field over the region of interest). The eddy current problem associated with the changing magnetic field can be solved by with either a set of steady-state AC problems or by actually time marching a DC transient where the coil currents follow the sinusoidal variations.

For a ship with a maximum roll angle $\theta_{\max}$ and a roll frequency f in a vertical magnetic field of density B_{vert} :



$$B_X(t) = B_{vert}\sin(\theta(t)) = B_{vert}\sin(\theta_{\max}\sin(2\pi ft)) \quad (1)$$

$$B_Y(t) = B_{vert}\cos(\theta(t)) = B_{vert}\cos(\theta_{\max}\sin(2\pi ft)) \quad (2)$$

Equations (1) and (2) may be expanded in the usual power series. For small angles manipulation and truncations leads to:

$$B_X(t) = B_{vert}\theta_{\max}\sin(\omega t) \quad (3)$$

$$B_Y(t) = B_{vert}(1 - \frac{\theta^2_{\max}}{4}) + \frac{\theta^2_{\max}}{4}\cos(2\omega t) \quad (4)$$

Therefore, for small maximum roll angles only three separate problems are solved:

(1) A static problem with a purely vertical field of B_{vert}

(2) An AC SS problem with a purely horizontal field of $B_{vert}\sin(\omega t)$; and,

(3) An AC SS problem with a purely vertical field of $B_{vert}\cos(2\omega t)$

Once these solutions are obtained, the total solution is the superposition of the real parts. E.g., the field at a point (xp,yp,zp) at time, t is:

$$B_x(xp,yp,zp;t)=\left[(1-\frac{\theta^2{}_{\max}}{4})B_x^{vert}(xp,yp,zp;0)+\theta_{\max}\operatorname{Re}\{B_x^{horiz}(xp,yp,zp;t)\}+\frac{\theta^2{}_{\max}}{4}\operatorname{Re}\{B_x^{vert}(xp,yp,zp;t\}\right]$$

where

$$\operatorname{Re}\{B_x^{horiz}(xp,yp,zp;t)\}=\operatorname{Re}\{|B_x^{horiz}(xp,yp,zp)|\,e^{i(\omega t+\varphi_x^{horiz}(xp,yp,zp))}\}$$ with

$$|(xp,yp,zp)|=\sqrt{\operatorname{Re}\{B_x^{horiz}(xp,yp,zp)\}^2+\operatorname{Im}\{B_x^{horiz}(xp,yp,zp)\}^2}$$ and

$$\varphi_x^{horiz}(xp,yp,zp)=\tan^{-1}\left(\operatorname{Im}\{B_x^{horiz}(xp,yp,zp)\}^2/\{B_x^{horiz}(xp,yp,zp)\}^2\right)$$

Or

$$\operatorname{Re}\{B_x^{horiz}(xp,yp,zp;t)\}=|B_x^{horiz}(xp,yp,zp)|\cos(\omega t\pm\varphi_x^{horiz}(xp,yp,zp))$$

Therefore:

$$B_x(xp,yp,zp;t)=(1-\frac{\theta^2{}_{\max}}{4})B_x^{vert}(xp,yp,zp;0)+\theta_{\max}|B_x^{horiz}(xp,yp,zp)|\cos(\omega t\pm\varphi_x^{horiz}(xp,yp,zp))+\frac{\theta^2{}_{\max}}{4}|B_x^{vert}(xp,yp,zp)|\cos(2\omega t\pm\varphi_x^{vert}(xp,yp,zp)) \quad (5)$$

Similar expressions occur for the other components and variables such as current density, etc. Of course, the problems may be solved with a unit vertical field and then all results scaled to B_{vert}. Also, depending on the definition of the phase angle, φ, there may be a minus sign instead of a plus sign preceding the phase angle. In the example below, the – sign was necessary.

When comparing the results from (5) directly with a transient (ELEKTRA-TR) run there may be a time shift required since the AC solutions have no "start-up transient" and the TR run does.

EXAMPLE

Figure 1 shows a simple ship model rolling at 0.1 Hz in a vertical magnetic field of strength 1 A/m. The maximum roll angle is 5 degrees.

Figure 2 shows the horizontal component of the magnetic field signature 20 meters below the keel centered on the ship – i.e., x=0, y=-20, z=0. The curves are for a an ELEKTRA-TR run through 4 full cycles with an adaptive time step of 0.125 seconds and a maximum time step error of 1%. The symbols arise from equation (5) with the three AC steady runs described above.

Figure 3 shows the vertical component of the signature at the same point. The TR and AC curves are as described above. There is a constant offset between the two. If the 0 frequency (DC) case is scaled by 1.0015 (i.e., increased 0.15%) this scaled AC and TR curves coincide after the initial transient dies out.

As mentioned above, for this case, the phase angles were taken as negative.

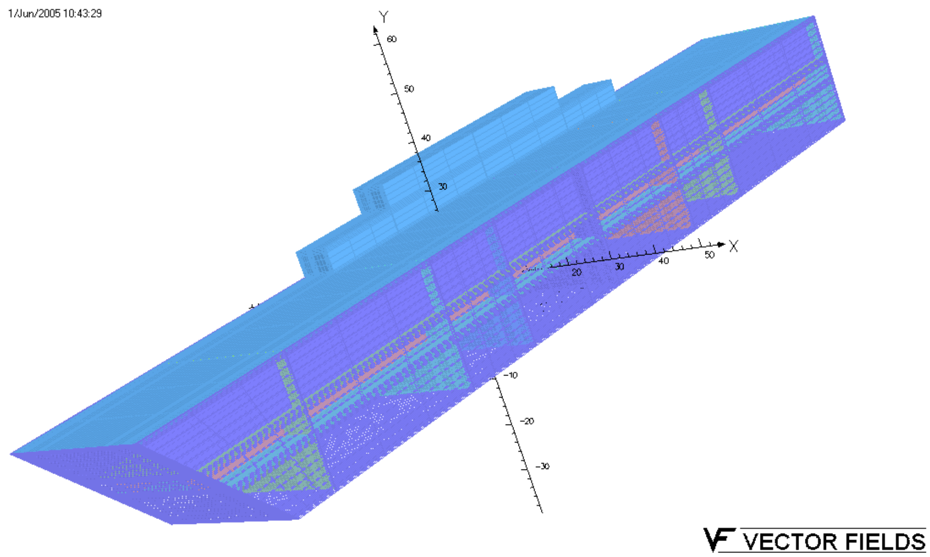


Figure 1 – Model of a simple asymmetric ship – translucency used to show internal structure.

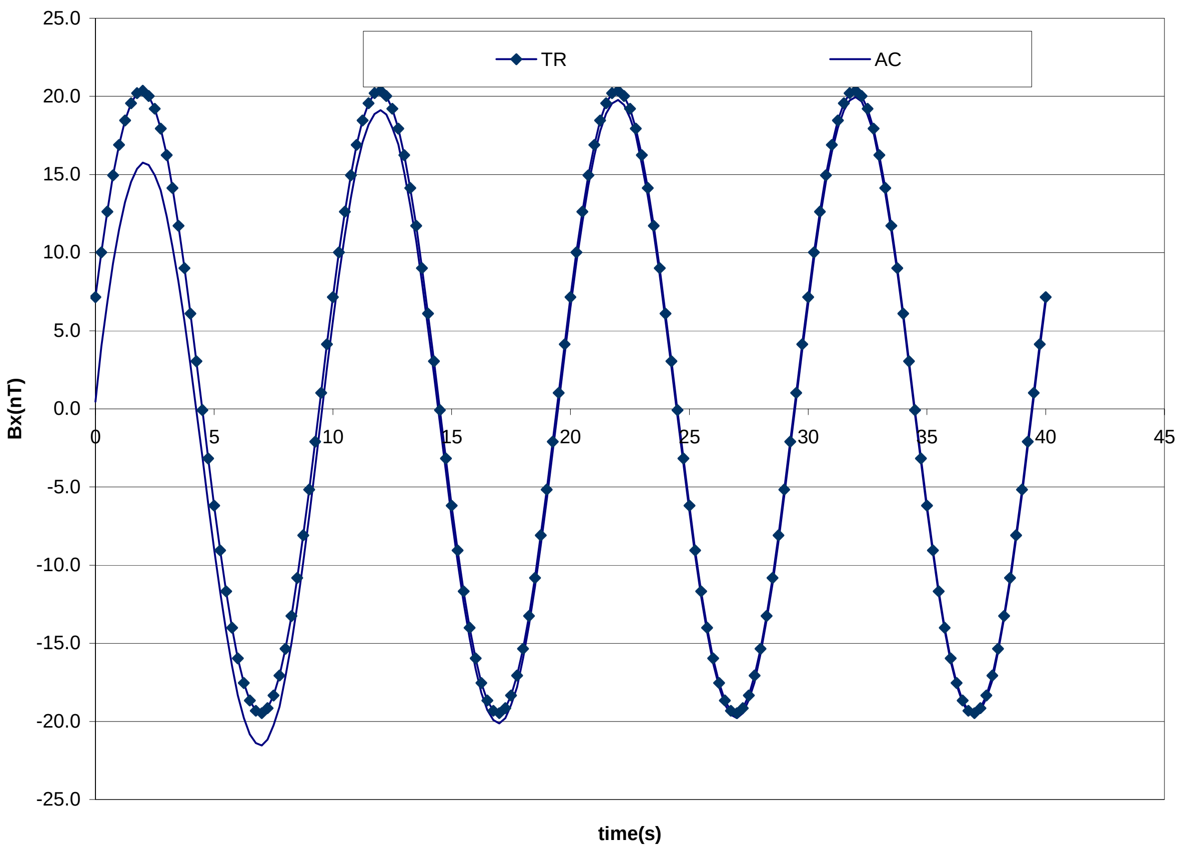


Figure 2 – Horizontal Component of the signature at (0,-20,0).

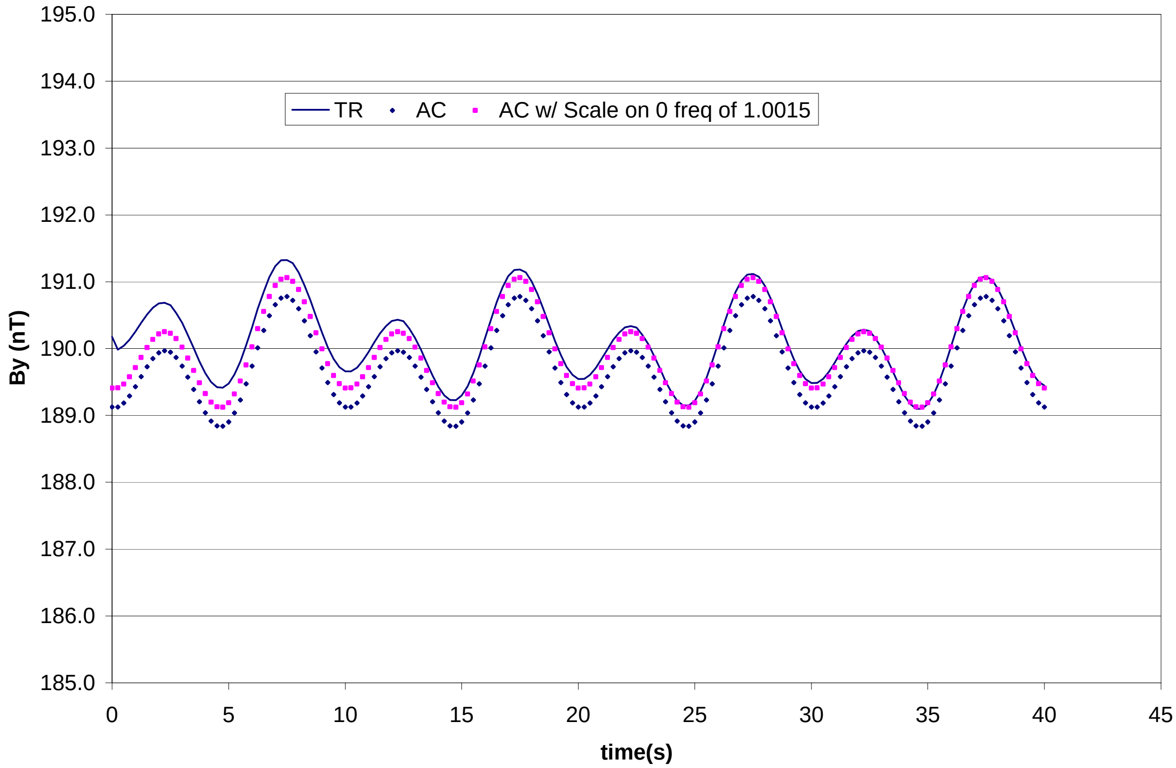


Figure 2 – Vertical Component of the signature at (0,-20,0).